

Name: _____

Date: _____

A2CC - Rational Exponents

If $x \geq 0$ and n is a positive integer then:

(1) $x^{\frac{1}{n}} = \sqrt[n]{x}$

(2) $x^{\frac{m}{n}} = (\sqrt[n]{x})^m = \sqrt[n]{x^m}$

(3) $x^{-\frac{m}{n}} = \frac{1}{(\sqrt[n]{x})^m} = \frac{1}{\sqrt[n]{x^m}}, \text{ provided that also } x \neq 0$

Examples:

1. $256^{\frac{1}{2}}$

2. $8^{\frac{1}{3}}$

3. $(\frac{27}{64})^{\frac{1}{3}}$

4. $8^{\frac{2}{3}}$

5. $32^{-\frac{2}{5}}$

6. $6^{\frac{3}{2}}$

7. $\frac{12^0}{9^{-\frac{3}{2}}}$

8 a. $(-27)^{\frac{1}{3}}$

b. $-27^{\frac{1}{3}}$

9. $(\frac{-8a^3}{b^{-6}})^{\frac{2}{3}}$

Rewrite each of the following using fractional exponents.

10. a. $5\sqrt{x}$

b. $\sqrt[3]{5x^2}$

c. $\sqrt[3]{(5x)^2}$

d. $\sqrt[5]{x^4 y^3}$

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| 11. $(2 \times 8)^{\frac{1}{4}}$ | 12. $5(81)^{\frac{1}{4}}$ | 13. $-4(1,000)^{\frac{1}{3}}$ | 14. $49^{\frac{3}{2}}$ |
| 15. $8^{\frac{5}{3}}$ | 16. $27^{\frac{2}{3}}$ | 17. $10,000^{\frac{1}{4}}$ | 18. $32^{\frac{2}{3}}$ |
| 19. $9^{-\frac{1}{2}}$ | 20. $8^{-\frac{1}{3}}$ | 21. $100^{-\frac{1}{2}}$ | 22. $125^{-\frac{2}{3}}$ |
| 23. $3^{\frac{1}{2}} \times 3^{\frac{3}{2}}$ | 24. $5^{\frac{1}{3}} \times 5^{\frac{2}{3}}$ | 25. $7^{\frac{1}{4}} \times 7^{\frac{3}{4}}$ | 26. $4 \times 4^{\frac{1}{2}}$ |
| 27. $32 \times 32^{\frac{1}{2}}$ | 28. $2^{\frac{1}{3}} \times 8^{\frac{1}{3}}$ | 29. $12^{\frac{1}{3}} \div 12^{\frac{2}{3}}$ | 30. $3^{\frac{2}{3}} \div 3^{\frac{1}{3}}$ |
| 31. $4^{\frac{1}{3}} \div 4^{\frac{2}{3}}$ | 32. $125^{\frac{2}{3}} \div 125^{\frac{1}{3}}$ | 33. $4^0 + 4^{-\frac{1}{2}}$ | 34. $9^{-2} + 9^{\frac{1}{2}}$ |
| 35. $2[(3)^{-2} + (4)^{-2}]^{-\frac{1}{2}}$ | 36. $(2.3 \times 10^{-1})(5.2 \times 10^{-3})$ | 37. $\frac{(2(3)^2 + \frac{1}{3^{-2}})^{\frac{1}{2}}}{6(2 + \frac{1}{4})^{-\frac{1}{2}}}$ | |

In 38–57, write each radical expression as a power with positive exponents and express the answer in simplest form. The variables are positive numbers.

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| 38. $\sqrt{7}$ | 39. $\sqrt{6}$ | 40. $\sqrt[3]{12}$ | 41. $\sqrt[3]{15}$ |
| 42. $\sqrt[4]{3}$ | 43. $\sqrt[5]{2^3}$ | 44. $(\sqrt[5]{9})^4$ | 45. $\frac{1}{(\sqrt{5})^3}$ |
| 46. $\sqrt{25a}$ | 47. $\sqrt{49x^2}$ | 48. $\sqrt{64a^3b^6}$ | 49. $\frac{1}{2}\sqrt{18a^6b^2}$ |
| 50. $\sqrt{9a^{-2}b^6}$ | 51. $\sqrt{\frac{3a}{4b}}$ | 52. $\sqrt[3]{27a^3}$ | 53. $\sqrt[4]{64x^5}$ |
| 54. $\frac{1}{\sqrt[5]{xyz^5}}$ | 55. $\sqrt{\frac{9a^{-2}}{4b^4}}$ | 56. $\sqrt[10]{\frac{10^{15}x^{20}}{y^5}}$ | 57. $\sqrt[8]{\sqrt[4]{a} \cdot \sqrt[4]{b^7}}$ |

In 58–73, write each power as a radical expression in simplest form. The variables are positive numbers.

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| 58. $3^{\frac{1}{2}}$ | 59. $5^{\frac{1}{3}}$ | 60. $6^{\frac{1}{4}}$ | 61. $9^{\frac{1}{5}}$ |
| 62. $5^{\frac{2}{3}}$ | 63. $12^{\frac{1}{4}}$ | 64. $6^{\frac{3}{5}}$ | 65. $\frac{1}{5^{\frac{1}{2}}}$ |
| 66. $(x^{13})^{\frac{1}{7}}$ | 67. $(25x^2y)^{\frac{1}{2}}$ | 68. $(50ab^4)^{\frac{1}{2}}$ | 69. $(16a^5b^6)^{\frac{1}{4}}$ |
| 70. $\frac{(x^5y^6)^{\frac{1}{2}}}{z^{-\frac{1}{2}}}$ | 71. $\frac{5a^{\frac{1}{2}}}{4^{\frac{1}{3}}}$ | 72. $(\frac{-32x^{10}}{y^4})^{\frac{1}{5}}$ | 73. $\frac{8^{\frac{1}{2}}a^{\frac{1}{3}}b^{\frac{1}{5}}}{(27c^4)^{\frac{1}{3}}}$ |

In 74–82, write each expression as a power with positive exponents in simplest form.

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| 74. $(\frac{2a^{\frac{1}{2}}}{3a^{\frac{1}{3}}})^6$ | 75. $(\frac{x^2y}{3x^4b^2})^{\frac{2}{3}}$ | 76. $(\frac{4a^4b^6}{25a^{-1}b})^{\frac{1}{2}}$ |
| 77. $(\frac{8a^2z^6}{27x^9a^{-4}z^{-1}})^{\frac{1}{3}}$ | 78. $\sqrt{x^2y} \cdot \sqrt{x^4y^3}$ | 79. $\frac{\sqrt[5]{a^5}}{\sqrt[3]{a^3}}$ |
| 80. $\frac{\sqrt[3]{11x^5y^4}}{\sqrt{2x^5y^3}}$ | 81. $\frac{\sqrt[3]{48xy^2}}{\sqrt[4]{6x^3y^4}}$ | 82. $(\sqrt{2xy^2})(\sqrt[3]{16x^2y})$ |

83. Verify that the laws for power of a product and power of a quotient are true for the following examples. In each example, evaluate the left side using the rules for radicals and the right side using the rules for fractional exponents:

a. $(\sqrt[3]{27} \cdot \sqrt[3]{3})^2 \stackrel{?}{=} (27^{\frac{1}{3}} \cdot 3^{\frac{1}{3}})^2$ b. $(\frac{\sqrt{3}}{\sqrt{9}})^3 \stackrel{?}{=} (\frac{3^{\frac{1}{2}}}{9^{\frac{1}{2}}})^3$